Lesson 4 — Solving Multiplication Equations

Some equations that include multiplication can be solved mentally. For example, to solve $9q = 36$, think "Nine times what number is 36?" Since $9 \cdot 4 = 36$, $q = 4$.

When equations that include multiplication cannot be solved mentally, you can find the value of the variable by multiplying both sides of the equation by the reciprocal of the coefficient.

EXAMPLE 1   Solve $3w = 57$ for w.

Step 1	Write the equation.	$3w = 57$
Step 2	Multiply both sides of the equation by $\frac{1}{3}$, the reciprocal of 3.	$(\frac{1}{3})3w = 57(\frac{1}{3})$
Step 3	Simplify.	$(\frac{1}{3})\frac{3}{1}w = \frac{57}{1}(\frac{1}{3})$
		$\frac{3}{3}w = \frac{57}{3}$
		$w = 19$
Step 4	Check.	$3 \cdot 19 = 57$

Multiplying by $\frac{1}{3}$ is the same as dividing by 3. In general, dividing by a number n is the same as multiplying by $\frac{1}{n}$. This fact gives you a choice—you can solve equations that include multiplication by dividing or by multiplying by the reciprocal of the coefficient.

EXAMPLE 2   Find the value of the variable in the expression $5c = 125$.

Divide each side by 5.

$$5c = 125$$
$$\frac{5}{5}c = \frac{125}{5}$$
$$c = 25$$

Check. $5 \cdot 25 = 125$

Multiply each side by $\frac{1}{5}$.

$$5c = 125$$
$$(\frac{1}{5})5c = 125(\frac{1}{5})$$
$$\frac{5}{5}c = \frac{125}{5}$$
$$c = 25$$

Check. $5 \cdot 25 = 125$

Some algebra equations involve multiplication with fractions. To solve these equations, multiply both sides of the equation by the reciprocal of the fraction.

Always check your work. Replace the variable with your answer and solve the problem.

EXAMPLE 1 Solve $\frac{1}{4}h = 6$ for h.

Step 1 Write the equation. $\frac{1}{4}h = 6$

Step 2 Multiply both sides of the equation by $\frac{4}{1}$, the reciprocal of $\frac{1}{4}$. $\left(\frac{4}{1}\right)\frac{1}{4}h = 6\left(\frac{4}{1}\right)$

Step 3 Simplify.
$$\left(\frac{4}{1}\right)\frac{1}{4}h = \frac{6}{1}\left(\frac{4}{1}\right)$$
$$\left(\frac{4}{1}\right)\frac{1}{4}h = \frac{6}{1}\left(\frac{4}{1}\right)$$
$$\frac{4}{4}h = \frac{24}{1}$$
$$h = 24$$

Step 4 Check. $\frac{1}{4}(24) = 6$

EXAMPLE 2 Solve $-\frac{2}{3}m = 12$ for m.

Step 1 Write the equation. $-\frac{2}{3}m = 12$

Step 2 Multiply both sides of the equation by $-\frac{3}{2}$, the reciprocal of $-\frac{2}{3}$. $\left(-\frac{3}{2}\right)-\frac{2}{3}m = 12\left(-\frac{3}{2}\right)$

Step 3 Simplify.
$$\left(-\frac{3}{2}\right)-\frac{2}{3}m = 12\left(-\frac{3}{2}\right)$$
$$\left(-\frac{3}{2}\right)-\frac{2}{3}m = \frac{12}{1}\left(-\frac{3}{2}\right)$$
$$\frac{6}{6}m = -\frac{36}{2}$$
$$m = -18$$

Step 4 Check. $\left(-\frac{2}{3}\right)(-18) = 12$
$$\frac{36}{3} = 12$$

Rule To solve an equation that involves multiplication with fractions, multiply both sides of the equation by the reciprocal of the fraction.