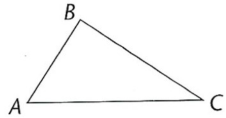
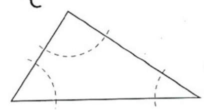
Lesson 3 Angle Measures in a Triangle

Any triangle is made up of three sides and three angles. The sides and angles of triangles share relationships that can be proven. For example, one way to discover the sum of the measures of the angles in any triangle is to do this experiment.

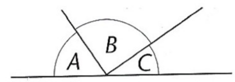
1. Draw any triangle.



2. Using scissors, cut off each angle of the triangle.



3. Place the angles adjacent to each other along a straight line.



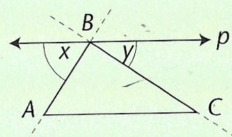
Since the exterior sides of the angles form a supplementary angle and the measure of a supplementary angle is 180°,

$$m\angle A + m\angle B + m\angle C = 180°.$$

You can also prove this as a theorem. This is an example of a proof.

EXAMPLE 1 **Theorem:** The sum of the measures of the angles in any triangle is 180°.

Prove $m\angle A + m\angle B + m\angle C$ in $\triangle ABC = 180°$.



Step 1 $\triangle ABC$ is a triangle. Given.

Step 2 Draw Line $p \| AC$ through Point B. By construction.

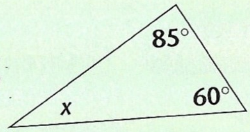
Step 3 AB is a transversal of p and AC, so $m\angle x = m\angle A$ because alternate interior angles are equal.

Step 4 BC is a transversal of p and AC, so $m\angle y = m\angle C$ because alternate interior angles are equal.

Step 5 $\angle x + \angle B + \angle y$ are adjacent and form a straight angle. Since the measure of a straight angle is 180°, $m\angle x + m\angle B + m\angle y = 180°$ and by substitution $m\angle A + m\angle B + m\angle C = 180°$.

You can use this angle-sum theorem to solve problems like the following example.

EXAMPLE 2 One angle in a triangle measures 60°. Another measures 85°. What is the measure of the third angle?



Step 1 Since the sum of the measures of the angles in any triangle is 180°, $x + 60° + 85° = 180°$.

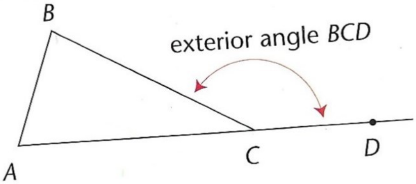
Step 2 Solve $x + 60° + 85° = 180°$ for x.

$$
\begin{aligned}
x + 60° + 85° &= 180° \\
x &= 180° - 60° - 85° \\
x &= 35°
\end{aligned}
$$

Step 3 Check.

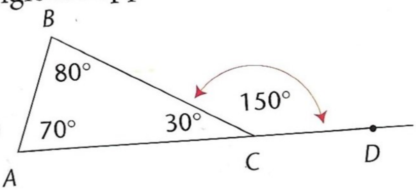
$$
\begin{array}{r}
60° \\
85° \\
\underline{+\ 35°} \\
180°
\end{array}
$$

Other relationships exist in any triangle. For example, an exterior angle of a triangle is formed by extending one side of a triangle at any vertex.



$\angle BCD$ is an exterior angle.

Any exterior angle of a triangle is supplementary to the adjacent interior angle.



$\angle BCA$ is supplementary to $\angle BCD$, so

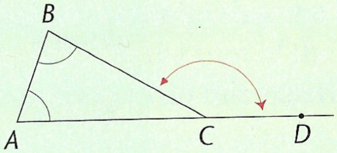
$$
\begin{aligned}
m\angle BCA + m\angle BCD &= 180° \\
30° + m\angle BCD &= 180° \\
m\angle BCD &= 180° - 30° \\
m\angle BCD &= 150°
\end{aligned}
$$

In this example, note that the sum $m\angle A + m\angle B$ is equal to $m\angle BCD$. In other words, the sum of the measures of the two non-adjacent interior angles in any triangle is equal to the measure of the exterior angle.

Here is another example of a proof.

EXAMPLE 3 **Theorem:** The measure of any exterior angle of a triangle is equal to the sum of the measures of the two non-adjacent interior angles.

In $\triangle ABC$, prove $m\angle A + m\angle B = m\angle BCD$.



Step 1 $\triangle ABC$ is a triangle. Given.

Step 2 $m\angle A + m\angle B + m\angle C = 180°$. By theorem, the sum of the measures of the angles in any triangle is 180°.

Step 3 $m\angle C + m\angle BCD = 180°$. The sum of the measures of supplementary angles is 180°.

$$m\angle A + m\angle B + m\angle C = 180° \qquad m\angle C + m\angle BCD = 180°$$

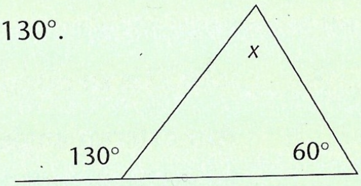
$$m\angle A + m\angle B = 180° - m\angle C \qquad m\angle BCD = 180° - m\angle C$$

$$m\angle A + m\angle B = m\angle BCD$$

Quantities equal to the same quantity are equal to each other.

You can use this theorem to solve problems like the following examples.

EXAMPLE 4 Given $\triangle ABC$, find $m\angle x$.

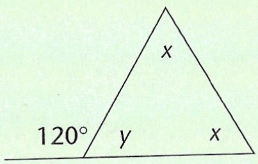


Step 1 By previous theorem, $60° + m\angle x = 130°$.

Step 2 Solve $60° + m\angle x = 130°$ for x.

$$60° + m\angle x = 130°$$
$$m\angle x = 130° - 60°$$
$$m\angle x = 70°$$

EXAMPLE 5 In this triangle, $m\angle x = m\angle y$. The exterior angle measures 120°. Find $m\angle x$ and $m\angle y$.



Step 1 Find $m\angle y$. Since $\angle y$ and the exterior angle are supplementary, the sum of their measures is 180°.

$$120° + y = 180°$$
$$y = 180° - 120°$$
$$y = 60°$$

Step 2 Find $m\angle x$.

$$y + x + x = 180°$$
$$60° + x + x = 180°$$
$$x + x = 180° - 60°$$
$$2x = 120°$$
$$x = 60°$$

Step 3 Check. $60° + 60° + 60° = 180°$

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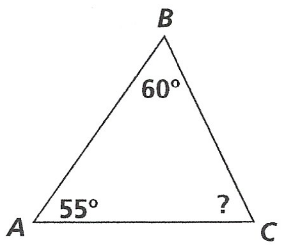
Angle Measures in a Triangle

EXAMPLE

Theorem

$m\angle A + m\angle B + m\angle C = 180°$



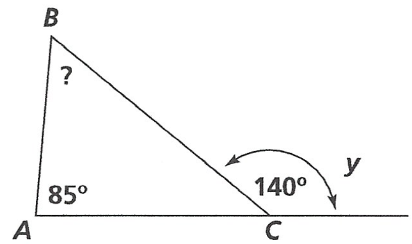
Find the measure of $\angle C$.

Let $n = m\angle C$.

$n + 60° + 55° = 180°$ Solve: $n = 65°$

Theorem

$m\angle A + m\angle B = m\angle y$

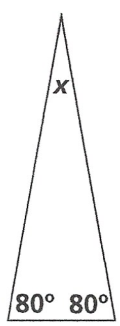


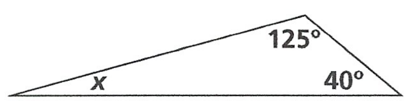
Find the measure of $\angle B$.

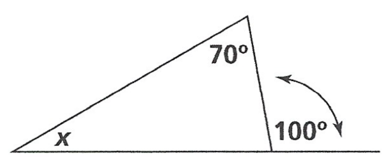
Let $k = m\angle B$.

$85° + k = 140°$ Solve: $k = 55°$

Directions Find $m\angle x$ in each triangle.



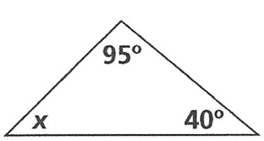


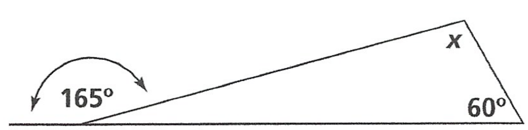


1. __________

2. __________

3. __________





4. __________

5. __________